A Non-linear Analytical Approach to Single Cells

In Cumulative Frequency Tables

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ABSTRACT

Having established clear evidence for a form of learning that is transformational and non-linear (Powell & Shklov, 1992), the authors herewith present a non-linear way to consider single cell frequency data in social-science settings. This paper describes the procedure in detail and gives several instances of its application in diverse settings.

The Problem

These two researchers have, for some time, been investigating non-linear approaches to observational data. The initial problem arose because they were trying to find some way to examine, in multiple-choice tests, the patterns in which students' answers (whether right or wrong) change with learning.

As background to this problem there are two possibilities. The first possibility is that all wrong answers are blind guesses and that students will change from any wrong answer to the right one with appropriate instruction. In this case the current practice of considering only right answers when observing student behavior changes for quality control would be entirely appropriate. There would be no meaningful information to be found among the wrong answers.

The second possibility, which comes from Powell's teaching experience and from Piaget's many years of clinical observation (see: Flavell, 1963), is that answers change in some sort of sequential order that is dependent upon how students interpret the test questions. Their interpretations, in turn, could be dependent upon their current style of thinking and their current depth of intellectual maturity.

In this case, learning would display transformational properties (systematic answer changes) in addition to cumulative properties (increasing numbers of correct answers). There could be considerable meaningful information in *all answers*, not merely the right ones.

To look at change, it is necessary to give the same test more than once. When this is done, it becomes possible to compare the answers selected upon the first administration to those selected on the second administration and build frequency tables that summarize these selections. In some cases, the students may select the same answer on both administrations, or they may change from one answer to another within each item.

If the first possibility (that students guess blindly until they know the answer) is correct, the typical observation should be changes to the right answer from any wrong one followed by systematic repetition of the right answer over time. No other combination of selection should be statistically significant beyond a chance level.

If the second possibility is correct, then a much more complex pattern of answer stability and answer change should become evident as students transform their thinking with increased maturity.

From a statistical perspective the problem is to determine the cumulative probability of a single observed frequency in a cross-tabulated frequency table with either $P \geq 0.95$ or $P \leq 0.05$ when the table is not diagonally symettrical.

The Approach

In a four-option multiple-choice test there will be a frequency matrix for each item containing 16 cells The one direction may be used to represent the responses from the first administration of the test and the other direction to represent the responses from the second administration. In this case each cell will be the frequency of the event of joint selection for each pre-post response pair. Those students who, for some reason, omitted making a choice at either of two administrations may be dropped from consideration because these data will not add to the information about the dynamics of answering.

With these frequency tables, there are four pieces of information that can be considered firm for each cell frequency. These are:

> The number of times the members of the sample chose both members of the pair of responses being considered (the *observed* frequency),

The number of times this group chose the first member of the pair on the first administration, (the column total)

The number of times this group of students chose the second member of this pair on the second administration (the row total), and

The total number of observations in the entire frequency table (table total).

In mathematical terms, the values being considered are the cell frequency, its associated row and column sums, and the table sum.

When these matrices are displayed, they look something like the typical contingency table, except that the concern here is with each cell instead of the overall departure of the entire matrix from a χ^2 distribution. These matrices are not diagonally symmetrical because events such as A $\Rightarrow$ is not equivalent to B $\Rightarrow$ A and either event is mutually exclusive to the other.

Insert Table 1 about here

Suppose, as illustrated in Table 1.a, that we have a joint choice of 11, a first variable frequency of 15, and a second variable frequency of 19. If the group size is 23, the expected joint choice frequency is $(15 \times 19)/23 = 12.39$. In this case, 11 is a lesser value than this expectation but may not be enough below this expected value to conclude that choosing one option implies the rejection of the other. If the group size is 90, as illustrated in Table 1.b, the expected frequency is $(15 \times 19)/90 = 3.17$. Is 11 enough larger than this expectation to indicate an outlier?

After investigation of the literature and numerous simulations of existing proposals to find a way to put a probability value on the cell in question with known marginal sums and

any total frequency, it became evident that a more sophisticated approach was required for an effective analytical solution to this interpretive problem. One such resolution is reported in this paper.

Because a single-dimensional array can be treated as a binomial distribution, these researchers tried using the multinomial distribution for the two dimensional arrays being studied. They developed an adaptation of this approach that involves collapsing these tables to two-by-two tables around each cell of concern. They then calculated the probability of observing that frequency or less with the other three values held constant. It is the purpose of this paper to provide the details of this procedure, with examples from different types of qualitative research.

The Procedure

Step 1.

This procedure begins with an $m \times n$ matrix of frequencies O_{ij} as follows:

$$
\begin{array}{ccccc}
O_{11} & O_{12} & \ldots & O_{1n} & R_1 \\
O_{21} & O_{22} & \ldots & O_{2n} & R_2 \\
\ldots & \ldots & \ldots & \ldots & \ldots \\
O_{m1} & O_{m2} & & O_{mn} & R_m \\
 & & & & \\
C_1 & C_2 & \ldots & C_n & N
\end{array}
$$

where: $$R_i = \sum_{j=1}^{n} O_{ij} \,, \qquad (1)$$

$$C_j = \sum_{i=1}^{m} O_{ij} \,, \qquad (2)$$

and: $$N = \sum_{i=1}^{m} \sum_{j=1}^{n} O_{ij} \,. \qquad (3)$$

Step 2

The next step is to collapse this matrix around any cell O_{ij} . This step produces a 2 × 2 matrix, with the observed frequency of O_{ij} being *o*, of the sort:

o	f	R'_1	where:	$f = R'_1 - o$,
g	h	R'_2		$R'_2 = N - R'_1$,
C'_1	C'_2	N		$g = C'_1 - o$,
				$C'_2 = N - C'_1$,
			and	$h = N - (o + f + g)$.

In words, *f* is the frequency of the remainder of the row, *g* is the frequency of the remainder of the column and *h* is the residual frequency of the total table. Tables 2 and 3 give a numerical example of these first two steps in this procedure, beginning with the 4 × 4 frequency matrix, such as:

Insert Table 2 about here

By choosing to look at O_{22} (with a frequency of $o = 33$; the repeated choice of the correct answer as shown in the box in Table 2) the collapsed matrix becomes:

Insert Table 3 about here

In order to determine the probability that *o* would be 33 by chance alone, it is necessary to find two cumulative probabilities. First, it is necessary to find the cumulative probability for the range of all possible values in that cell with the three marginal values kept constant. Second, there is the need to find the cumulative probability for all single tables that have a value of 33 or less.

To achieve this end, it is necessary to find the smallest possible value and the largest possible value that *o* can achieve within these marginal constraints.

Since these values are all frequencies, and the least possible frequency is zero (0), the least possible value that *o* can possess is when *either the frequency value of o or of h is zero (0)*. Similarly, the greatest possible value that cell O_{ij} can contain occurs when *either the frequency of f or of g is zero (0)*.

Mathematically:

$o_{(\min)} = 0$, if $o \leq h$; otherwise $o_{(\min)} = o - h$ *(4)*

$o_{(\max)}$ = the lesser of R'_1 and C'_1 *(5)*

In the present case, the minimum value is 0 (zero) and the maximum value is 46. It is theoretically possible to have a matrix wherein the minimum possible value is not 0 (zero).

Step 3

The multinomial function is applied to the range of all possible values from the minimum (0) to the maximum (46), adding these partial probabilities to find the total possible probability (P_t) of events within these constrained conditions. The equation becomes:

$$P_t = \sum_{t = o_{(\min)}}^{o_{(\max)}} \frac{N!}{o_u!\, f_u!\, g_u!\, h_u!} \cdot (p_1)^{o_u} (p_2)^{f_u} (p_3)^{g_u} (p_4)^{h_u} \qquad (6)$$

In this process, the value of *u* is the incremental value across the range from $o_{(\min)}$ to $o_{(\max)}$.This means that the values of o_u and h_u will increase by one (1) and of f_u and g_u will decrease by one (1) until the range has been completed.

The values of p_1, etc. are constant. These are the probabilities for each of the four cells that are determined by constraining the calculations using the marginal totals. These constraints are imposed because we are assuming that the marginal proportions are the best estimates available for the actual distribution of these particular choices in the total sample.

Therefore these probabilities are the product of the joint proportions for each cell as follows:

$$p_1 = R'_1\, C'_1/N^2 \tag{7}$$

$$p_2 = R'_1\, C'_2/N^2 \tag{8}$$

$$p_3 = R'_2\, C'_1/N^2 \tag{9}$$

$$p_4 = R'_2\, C'_2/N^2 \tag{10}$$

Also:

$$f_u = R'_1 - o_u \tag{11}$$

$$g_u = C'_1 - o_u \tag{12}$$

and $$h_u = N - (o_u + f_u + g_u) \tag{13}$$

In this particular example, the values of p_1, p_2, p_3, and p_4 for o are 0.182, 0.254, 0.236, and 0.328 respectively, which adds to 1.000, for these four probabilities.

Step 4

The next step is to find the cumulative probability for the observed frequency or less (P_c), this time iterating from $o_{(\text{min})}$ to o using the same equation (6) and control parameters (7- 13) in step 3, as follows:

$$P_c = \sum_{c = o_{(\text{min})}}^{o} \frac{N!}{o_u!\, f_u!\, g_u!\, h_u!} \cdot (p_1)^{o_u} (p_2)^{f_u} (p_3)^{g_u} (p_4)^{h_u} \tag{14}$$

The difference between these two calculations being that equation *(6)* increments to $o_{(max)}$ from $o_{(min)}$ whereas equation *(14)* stops incrimination at the observed value of *o*.

Step 5

The desired cumulative probability for the observed cell frequency can now be determined. The cumulative probability for all possible events within these constrained conditions is P_t from equation *(6)* and the cumulative probability within these same constraints for the observed frequency or less is P_t from equation *(14)*. Thus the desired probability is the ratio of these two partial probabilities, or:

$$P_o = P_c \; / P_t. \qquad (14)/(6) \qquad (15)$$

In this example, the value of P_c is 0.0059 and of P_t is also 0.0059, so the probability of observing 33 choices of answer *o* with 46 and 48 overall choices and 110 observations, is P_o = 1.000. It appears, from these actual calculations, that the remaining 13 possible values (from 34 through 46) contribute less than one part in 20,000 of the total possible probability. hence the resulting probability is 1.000 when these partial probabilities are rounded to the fourth decimal place.

This probability represents the repeated choice of the *right* answer to this question at a particular age range (166 - 170 months). In this example thirty percent (33/110).of all answers are repeatedly right, while about two thirds ((33 + 33 / (46 + 48) or 66/ 94) of those who chose this answer on either administration chose it on both administrations. By chance alone the expectation would have been only 20.07 such choices within the constraints of the marginal values If their selections had been entirely random, this expectation would have dropped to 1/16 of the table total, or 6.875 events

The Results; A First Look

The next step is to consider the entire matrix given in Table 2 to see what this procedure can reveal. Here are the results of this one group of interactions across time within this one question. Table 4 gives the observed frequencies, expected frequencies and probabilities of all 16 interactions. It presents the frequencies that produced significantly high probabilities in enlarged bold print and the significantly low ones in reduced font size.

Insert Table 4 about here

Seven of the 16 interactions are significant; three high and four low. All the high ones are in the main diagonal, which means that there is a strong tendency in this age group for repeating the initial answer, *without regard as to whether it is right or wrong*.

There are also some very serious problems with these outcomes. One of the highest frequencies among the off-diagonal cells is O_{21} contains 8 joint selections but is classified as a significantly low probability of occurrence. The same event can be seen, to a greater or lesser degree for all the significantly low selection frequencies. The *most frequent changes* from one answer to another seem to be *significantly low* whenever the same two responses *are significantly high in their repeated choices*.

Perhaps this problem comes the mutually exclusive nature of multiple-choice test answering. In this case, these low probabilities could be statistical artifacts. To test this hypothesis, a second-order calculation was performed. These probabilities were recalculated after making the main diagonal elements zero (0). Table 5 reports these results:

Insert Table 5 about here

The results of the analysis of the frequencies from Table 2 can now be summarized in Table 6.

Insert Table 6 about here

The four significantly low probabilities disappeared in this second order analysis. The two low ones with relatively high frequencies compared with other off-diagonal elements became significantly high. In addition, one that was almost significant also became significantly high. There are now 6 of the 16 cells in the table that are significantly high, accounting for 78 of the 110 responses (more than 70 %), and none significantly low. This outcome compares with 33 stable responses (30 %) to the right answer. By regarding statistically significant probabilities to be psychologically meaningful, these frequencies now account for nearly two and a half times as much information as the stable right answers by themselves provide (78 / 33).

The conclusion that can be drawn from these results is that, once this second order analysis has been applied, this procedure seems to effectively identify the transformative-answer patterns that appear to be present upon visual examination of the original 4×4 table. With such encouraging results, the final step is to apply this procedure to a larger data set to observe whether it performs as well in a more general applied context. The balance of this paper is devoted to giving three examples of its application as a demonstration of its effectiveness, or lack thereof, for the non-linear, non-traditional analysis of frequency tables

Applications of this procedure

1. Reconsidering wrong answers

The first presentation is a restatement of the results from the data for which this procedure was developed as reported in Powell and Shklov (1992). It comes from two administrations of the same test to 2810 subjects ranging in age from less than eight years of age to more than eighteen, with a five-month interval between administrations. The entire sample was then subdivided into 27 age levels of five months each. The reported age in months comes from the subjects' ages at the first administration. The example we used above was age level 166 - 170 (median 168).

These data come from item 18 of Gorham's (1956) *The Proverbs Test*. This item reads:

QUICKLY COME, QUICKLY GO (Easy come, Easy go).

a) Always coming and going and never satisfied (median age 14).

b) What you get easily does not mean much to you (**right answer**).

c) Always do things on time (median age 8).

d) Most people do as they please and go as they please (median age 10).

A more detailed discussion of the interpretations characterizing these answers is given elsewhere (Powell, 1977, Powell and Shklov, 1992) in the literature. In the earlier study, this test was given to 550 elementary school children (grades 3 through 8) and then a representative sample of them was interviewed to determine the reasoning behind their answer selection.

In brief, alternative "c" was chosen most frequently by 8 year-olds. Their reasoning showed that they had interpreted "Quickly come ..." as it represented their personal physical activities in their third grade classroom. They had many changes of activity in a school day,

all of which required that they “finish on time.” In Piaget’s terms they would appear to be still egocentric and intuitive in their thinking.

Alternative “d” was chosen most frequently by 10 year-olds, with their reasoning reports suggesting that they were looking for the best literal interpretation of the proverb. This change in reasoning would suggest that they were no longer egocentric and were thinking concretely instead.

Next in age sequence was alternative “a”, most commonly chosen by 14 year-olds. Their reasoning seemed to suggest that they were moving out of literal and concrete thinking into more figurative interpretations.

The right answer, “b,” was chosen most commonly by 16 year-olds, suggesting that either they had reached formal operations or were answering from memory because of personal familiarity with this proverb.

This present report uses the data from a later study. The purpose of this later study was to try to replicate the sequence just presented and to determine whether the changes of answer selection coincided with the possibility that a developmental sequence might be present.

The present discussion provides the outcomes from the two orders of analysis combined into one table for clarity and convenience. Table 7 gives this consolidation with the numbers in larger font and in bold face being the significantly high frequencies drawn from this procedure. The numbers in smaller font and the light box are the significantly low frequencies. No significantly low frequencies occurred in the main diagonal.

Insert Table 7 here

The most obvious observation is the huge discrepancy between the number of cells of significantly high frequency to those of significantly low frequency (200 to 3). Similarly the number of significantly high frequency cells to the total number of cells (200 to 432) shows a considerable departure from random expectations, suggesting that these observations are reflecting some underlying systematic aspect of behavior. In the random case the number of significantly high cells and the number of significantly low cells should be about equal.

Considering the amount of recovered information is also revealing. Knowing that the correct answer to this question is "B," adds additional insight. Nearly all of the stable choices of this answer between the two administrations of the test are significantly high, suggesting a stronger tendency to repeat this choice than any other. However this stability, which is similar to test-retest reliability, accounts for only 23.8 % (669 to 2810) of all the answers and 32 % (669 to 2112) of all the significantly high frequencies.

In contrast the total of all significantly high frequencies to all answers is 75.2 % (2112 to 2810). This procedure appears to be recovering a great deal more systematic answering than the repeated correct answers alone can supply. The results of the entire group are similar to the observations made with age level 166 - 170 months reported earlier.

Also illuminating is the patterns of stability and change that can be observed. One way of defining a sequence is to consider two significantly high frequencies in chronological order initiate a sequence and two non-significant or significantly low cells break the sequence. Figure 1 gives these observations. These results are rearranged to show the evident sequential patterns.

Insert Figure 1 here

In the first place, it may be noted that the sequences of significantly stable choices ($c \Rightarrow c$, $d \Rightarrow d$, ...) tend to follow the same sequential pattern found in the earlier study. In addition, the median points of the changes from one answer to another ($c \Rightarrow d$, $d \Rightarrow a$, ...) tend to fall between the solid commencement ages for the stable repeated choices of this next sequential alternative. Thus answers appear to display a *replicating developmental sequenc*e.

Of particular interest is the change from the right answer “b” to the highest sequential wrong answer “a” with the more mature students. This reverse trend seems to reflect a move away from convergent thinking such as a multiple-choice test typically requires.

It is also interesting to note that these data seem to suggest there may be at least three levels of achievement present in those who leave school as part of the main-stream. There are those who have consolidated upon literal thinking (alternative “d”), those who have either completed formal operations or who can give the right answer from memory (alternative “b”), and those who either remained at the level of personalized thinking or gone beyond formal operations (alternative “a”). There are also a large number of students who are in the transition stages among these several developmental levels when they graduate.

Of further interest is the observation that there appears to be a second pathway as well. In this case the order seems to be the reverse of the main stream, going from the right answer “b” at a young age toward either literal thinking “d” or non-logical thinking “c” by the time that they leave school. Some of these students on this second pathway appear to leave school before they graduate. It was found that those students who withdrew from school between the two administrations most commonly chose either “c” or “d.”

The appearance of more than one developmental pathway is itself an interesting psychological observation. It begs the question, “How many such pathways can be found using this procedure?” Further research will be needed to answer this question. The

observation that this reverse pattern may be detectable at as young an age as ten years is encouraging. In combination these observations seem to indicate that using right answers as a criterion for academic success may be necessary but might be an insufficient measure of academic progress. The students' reasoning appears to determine the answers selected. It may be, as the interview data seems to suggest, that these transformational patterns among the answers could provide a window upon otherwise invisible learning processes.

This information is unavailable when tests are scored "right-wrong." Wrong-answer data is eliminated during scoring. Do these observations invalidate using this approach for test evaluation or educational quality assurance? Should we be using the information from *all* student answers instead?

2. Teachers' attitudes toward their profession

This second example being reported here is a fragment of a much larger study being conducted by the Consortium for Cross-Cultural Research in Education based at the School of Education of the University of Michigan in Ann Arbor. The study upon which this report is based was conducted in twelve countries, Allen Menlo being the coordinator. This fragment was prepared jointly between Powell and Collett (1990).

Secondary-school teachers were asked indicate, within brainstorming sessions, their sources for enthusiasm and discouragement toward their on-the-job experiences as teachers. Their comments were collected on wall charts, discussed, and then summarized.

Using a coding system, the written records of these discussions were reduced to seven categories, each of which could be designated as either as a source for enthusiasm (E) or discouragement (D). These categories were also classified as being controllable (C) by these teachers or beyond their immediate control (U) The categories were:

Self (C)

Students (C)

Others (C)

Self (U)

Students (U)

Climate (U)

Workload (U)

By looking at only three countries (Canada, Japan, and the U.S.A.) the multinomial procedure was used to consider similarities and differences among these countries on these fourteen categories. When the multinomial procedure is applied to the table the results were:

Insert Table 8 here

Two trends are observable. First, each country shows some differences from the other, with Canada and the U.S.A. more alike with each having eight of the fourteen significant probabilities in the identical direction. The directions of the other six were also equivalent. Japan differed from the other two countries by showing greater polarization and having an opposite view of the uncontrollable aspects of self and the classroom environment.

Second, there appears to be a trend wherein the sources for enthusiasm come from the elements of teaching over which teachers have control, and of discouragement from those elements over which they have little control. Although these conclusions contain no surprises, the procedure seems to identify quite clearly both general trends and international differences among these data.

3. Psychological characteristics of contrasting groups

In the third study reported here, Carney (1992) presents some simulated data designed to illustrate a clustering technique to aid in interpreting counseling sessions. In this paper twelve people, all of whom are adult children of alcoholic families, are contrasted. Six are in recovery and six are in denial. Table 8 gives the frequencies of responses in seven different categories for each of these six clients.

Insert Table 9 about here

These categories were compared as to topic, such that “living in a storm,” for those in recovery, was seen to be similar to “sudden, nasty mood swings,” for those in denial. Six of the seven could be contrasted in this way and their frequencies of response rank-ordered. The resulting ρ (rho) was - 0.49 which is negative but not significant. The multinomial probabilities are also given in Table 9.

Only one of these six pairs of content variables showed statistically significant contrast. However, four of the remaining five show a trend toward a similar polarity. It would appear that people in denial, and in recovery tend to display opposite psychological attitudes based solely upon the frequencies of things about which they are willing to talk during counseling sessions.

Conclusions and Implications

In this paper three different contexts have been presented, each one having in common frequencies of some sort. In each case the multinomial procedure has either supported the conclusions that would be expected from the situations that these data represent, or they have cast new light on situations where more traditional statistical procedures appear to be insufficient if not inappropriate.

This statistical procedure seems to be a promising new tool for researchers who are uncomfortable with more traditional techniques or are finding that such techniques do not penetrate their data as deeply as they might wish. The large increase in explained variability that sometimes has been observed herein holds promise for drawing more detail from data than more traditional statistical techniques may supply. Whenever there may be reason to suspect a high complexity among the events contained in frequency-table data, this procedure could prove helpful.

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Figure labels

1. Change Patterns for the Entire Item 18 (N = 2810).

Table 1

An Illustration of the Impact of

Sample Size Upon the Possible

Meaningfulness of Observed Frequencies

a.

	Option X	All other Options	Totals
Option Y	11	4	15
All Others	8	0	8
Totals	19	4	23

Expected value for 11 is 12.39

b.

	Option X	All other Options	Totals
Option Y	11	4	15
All Others	8	67	75
Totals	19	71	90

Expected value for 11 is 3.17

Table 2

An Example of the Procedure

Using Multiple-choice Test Data

	Frequency of answers on Pretest				
Posttest	A	**B**	C	D	Totals
A	11	5	3	9	28
B	8	$\boxed{33}$	2	5	48
C	3	2	0	2	7
D	7	6	3	11	27
Totals	29	46	8	27	110

Source: Data for age levels 166 - 170 months from Table 4, Powell & Shklov (1992).

The correct answer to this question is “B.”

Table 3

The Results of Collapsing

Table 1 Around Cell O_{22}

Posttest	Column_2	Columns_{1+3+4}	Totals
Row_2	33	15	48
Rows_{1+3+4}	13	49	62
Totals	46	64	110

Table 4

Probabilities of the Observed Frequencies

from Table 1: A First Look

Row 1:				
Observed	**11**	5	3	9
Expected	7.38	11.71	2.04	6.87
p values	0.997	0.002	0.888	0.907
Row 2:				
Observed	8	**33**	2	5
Expected	12.65	20.07	3.49	11.78
p values	0.034	1.000	0.235	0.002
Row 3:				
Observed	3	2	0	2
Expected	1.85	2.92	0.51	1.72
p values	0.923	0.376	0.580	0.771
Row 4:				
Observed	7	6	3	**11**
Expected	7.12	11.29	1.96	6.62
p values	0.584	0.014	0.901	0.993

Note: Probabilities where $p \leq .95$ are shown in increased font and bold face; while probabilities $p \geq 0.05$ are shown in reduced font size.

Table 5

Second Order Analysis of Table

with the Frequencies in Main Diagonal Removed

Row 1:				
Observed	***	5	3	**9**
Expected	*.**	1.18	2.47	4.95
p values	*.***	0.846	0.805	0.998
Row 2:				
Observed	**8**	***	2	5
Expected	4.91	*.**	2.8	4.36
p values	0.989	*.***	0.624	0.778
Row 3:				
Observed	3	2	***	2
Expected	2.29	1.65	*.**	2.04
p values	0.851	0.796	*.***	0.674
Row 4:				
Observed	7	**6**	3	***
Expected	5.24	3.78	4.65	*.**
p values	0.922	0.968	0.839	*.***

Table 6

Summary of the Results

of Both Orders of Analysis From Table 1

	Frequency of answers on Pretest				
Posttest	A	**B**	C	D	Totals
A	$\boxed{11}$	5	3	$\boxed{9}$	28
B	$\boxed{8}$	$\boxed{33}$	2	5	48
C	3	2	0	2	7
D	7	$\boxed{6}$	3	$\boxed{11}$	27
Totals	29	46	8	27	110

Note: Frequencies with a probability $p \leq 0.95$ are shown in the boxes. There were no low-probability events remaining in the analysis of this table.

Table 7

The Results of the Analysis of the two Administrations

of Item 18 from The Proverbs Test

Significance of Test-Retest Within-Item Responses

Ages	**A to A**	A to B	A to C	A to D	B to A	**B to B**	B to C	B to D	C to A	C to B	**C to C**	C to D	D to A	D to B	D to C	**D to D**	Age Group Totals	Mean-ingful Count
< 96	**1**	0	1	**2**	0	0	0	**1**	0	1	3	**3**	**1**	**3**	**5**	3	24	16
98	0	1	1	1	2	0	**4**	1	1	1	2	**2**	1	1	2	**11**	31	17
103	**2**	**1**	0	0	1	0	0	**2**	2	3	**5**	**3**	5	3	**4**	7	38	17
108	**15**	6	4	**9**	**4**	**5**	3	2	**6**	$\boxed{0}$	**5**	**6**	5	**7**	**5**	**19**	101	81
113	**11**	1	3	5	2	0	1	3	4	0	4	**7**	3	**4**	**5**	10	63	27
118	5	2	7	**18**	**5**	**4**	3	3	**5**	1	**8**	3	**7**	**3**	**8**	10	92	58
123	**11**	3	0	**13**	**5**	**4**	2	2	4	0	**6**	**8**	**10**	**8**	**6**	17	99	71
128	16	**6**	2	**10**	7	4	3	4	6	4	2	4	**10**	3	**5**	11	97	31
133	**15**	5	1	**12**	5	**13**	**4**	6	4	0	**3**	**5**	**9**	**8**	4	**21**	115	90
138	9	5	3	**16**	**4**	**13**	1	0	3	4	**4**	3	**13**	7	3	7	95	50
143	**16**	**10**	3	**8**	**8**	**15**	0	**6**	1	**6**	**3**	3	**16**	5	**5**	12	117	93
148	**10**	**8**	4	5	**5**	**16**	3	**5**	1	3	2	**3**	**7**	5	4	**9**	90	63
153	**11**	3	2	**11**	**5**	**14**	1	5	**4**	2	0	3	4	**5**	**3**	**14**	87	67
158	10	**8**	0	**5**	**9**	**20**	2	**6**	4	2	**5**	2	10	6	**4**	5	98	57
163	**10**	**5**	1	**4**	4	**16**	1	**4**	**3**	1	1	1	**6**	1	**4**	**11**	73	63
168	**11**	**8**	3	7	5	**33**	2	**6**	3	2	0	3	**9**	5	2	**11**	110	78
173	**16**	**17**	3	7	6	**60**	**5**	**8**	1	3	**3**	2	**13**	17	5	**22**	188	144
178	**15**	17	3	**14**	5	**54**	**4**	**9**	2	7	**3**	3	**12**	**20**	3	**22**	193	153
183	9	**20**	2	**10**	**15**	**70**	2	**10**	4	5	1	2	10	**20**	3	**18**	201	163
188	**16**	**14**	$\boxed{0}$	**6**	**15**	**63**	4	5	1	4	**2**	1	5	**17**	**6**	**18**	177	157
193	**15**	**14**	$\boxed{0}$	**7**	**10**	**56**	**8**	5	0	3	0	**4**	**8**	**12**	1	**17**	160	151
198	7	**18**	4	10	**9**	**48**	1	**9**	1	2	**1**	3	**6**	**9**	2	**14**	144	114
203	**9**	**11**	1	4	**12**	**38**	2	**6**	0	**3**	0	1	7	7	3	**13**	117	92
208	3	**8**	0	1	**7**	**38**	**4**	**6**	0	0	0	**1**	3	**9**	1	**7**	88	80
213	**9**	**5**	1	2	**6**	**38**	1	**3**	0	**1**	**1**	0	1	**7**	2	**7**	84	77
218	3	4	1	**6**	4	**16**	**5**	3	**2**	**2**	1	0	**3**	2	1	**5**	58	39
>220	**3**	**5**	1	1	**5**	**35**	2	**4**	0	**1**	**1**	0	3	**4**	0	**5**	70	63
Totals	258	205	51	194	165	673	68	124	62	61	66	76	187	198	96	326	2810	2112
Totals	196	158	0	151	124	669	34	85	20	13	50	42	130	136	60	244	2112	

Source: Composite of Tables 4 & 5 from Powell & Shklov (1992).

Age level 168 was used for the calculations in the example.

Table 8

Attribution Study

Basic Data and Probabilities

Attribution Target		U. S. A. Enthusiasm f	p	U. S. A. Discouragement f	p	Japan Enthusiasm f	p	Japan Discouragement f	p	Canada Enthusiasm f	p	Canada Discouragement f	p
	Controllable:												
1.	Self	**46**	**1.00**	3	0.00	**52**	**0.99**	33	0.00	**76**	**1.00**	10	0.00
2.	Students)	**48**	**1.00**	16	0.00	**128**	**1.00**	26	0.00	**89**	**1.00**	11	0.00
3.	Others	2	0.93	1	0.33	**6**	**0.97**	2	0.13	5	0.85	3	0.38
	Uncontrollable:												
4.	Self	**7**	**0.98**	2	0.10	11	0.00	**41**	**1.00**	12	0.63	11	0.53
5.	Students	11	0.46	12	0.70	51	0.00	**104**	**1.00**	18	0.00	**44**	**1.00**
6.	Climate	63	0.00	**99**	**1.00**	**157**	**0.96**	138	0.02	78	0.00	**161**	**1.00**
7.	Workload	18	0.00	**55**	**1.00**	13	0.00	**87**	**1.00**	19	0.00	**48**	**1.00**
	Totals	196		188		418		431		298		288	
									Grand Total			1819	

BOLD: Frequencies with $p \leq 0.95$.

Standard size: Frequencies not significant.

Small: Frequencies with $p \geq 0.05$.

Table 9

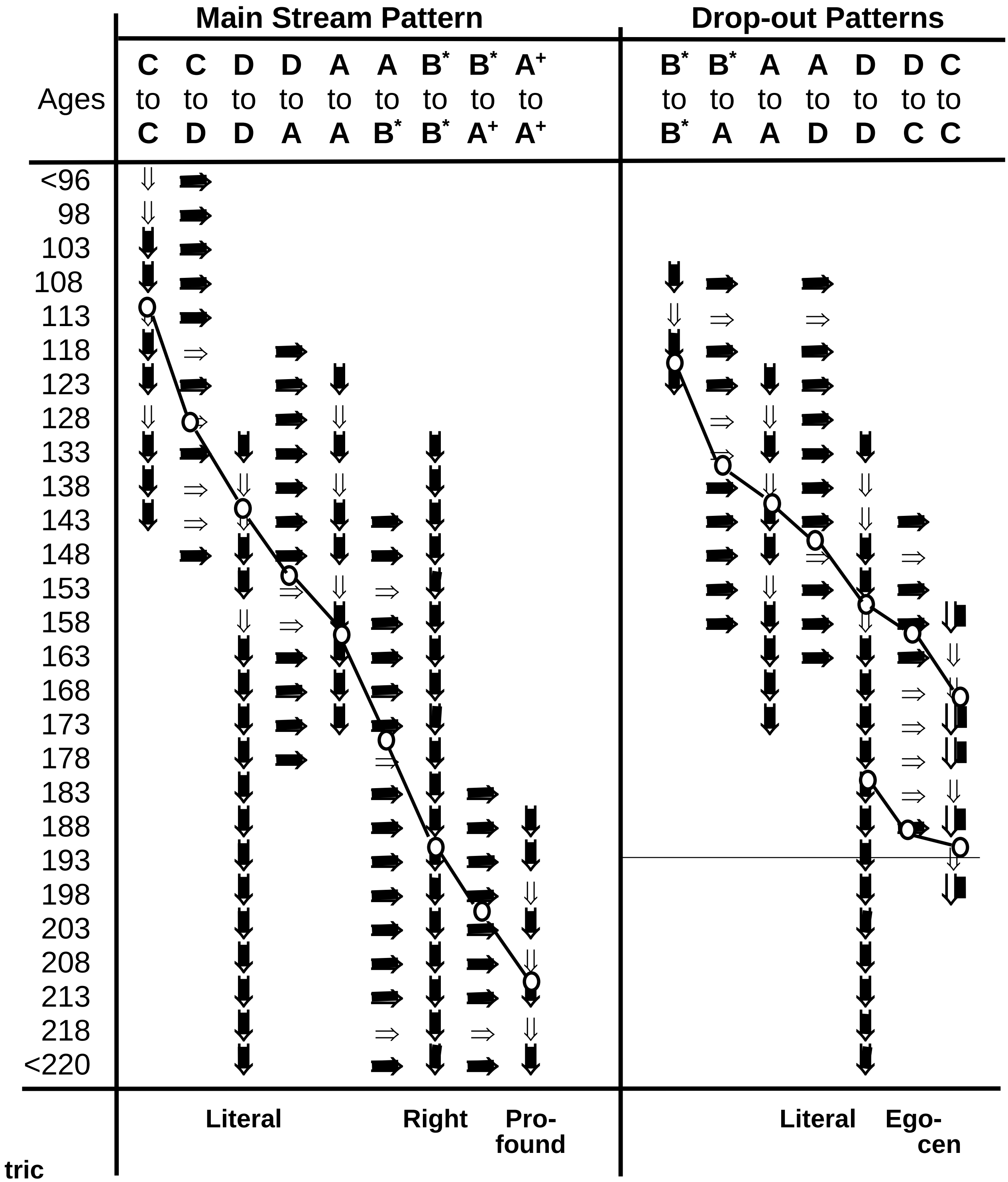
Frequencies of Responses to Various Topics

During Counseling Sessions

by Adult Children of Alcoholics

in Recovery of Denial

In Recovery			In Denial		
Topic of discussion	Frequency of references	Probab-ility	Topic of discussion	Frequency of references	Probab ility
1. Living in a storm	61	**0.996**	6. Sudden, nasty mood swings	23	0.009
2. Crazy making	56	0.884	5. Unfair preferences	29	0.172
3. Over-achieving	55	0.117	1. Don't do what dad did	45	0.921
4. Frozen feelings	52	0.584	3. Emotionally left home	33	0.512
5. Survivor guilt	47	0.159	2. Constant criticism	38	0.892
6. Not belonging	38	0.146	4. Physical violence	32	0.905



School leaving age (16 years old).

Significant selection frequencies ($p \leq 0.945$).

⇓ ⇒ Non-significant frequencies (sequence continuing).

Some of the identified developmental pathways.